Honors Algebra II
Trig Identities

Name _____
Hour _____ Date _____

Show that the left side of the equation equals the right side.

1. $\dfrac{\tan x}{1+\tan x} = \dfrac{\sin x}{\sin x + \cos x}$

$$= \frac{\frac{\sin x}{\cos x}}{1+\frac{\sin x}{\cos x}}$$

$$= \frac{\frac{\sin x}{\cancel{\cos x}}}{\frac{\cos x + \sin x}{\cancel{\cos x}}}$$

$$= \boxed{\frac{\sin x}{\cos x + \sin x}}$$

2. $\dfrac{\sec^4 x - 1}{\tan^2 x} = \tan^2 x + 2$

$$= \frac{(\sec^2 x + 1)(\sec^2 x - 1)}{\tan^2 x}$$

$$= \frac{\sec^2 x + 1 \cdot \cancel{\tan^2 x}}{\cancel{\tan^2 x}}$$

$$= \sec^2 x + 1$$

$$= 1 + \tan^2 x + 1$$

$$= \boxed{\tan^2 x + 2}$$

3. $\dfrac{\tan x}{1+\sec x} + \dfrac{1+\sec x}{\tan x} = 2\csc x$

$$= \frac{\tan^2 x + 1 + 2\sec x + \sec^2 x}{(1+\sec x)\tan x}$$

$$= \frac{\sec^2 x + 2\sec x + \sec^2 x}{(1+\sec x)\tan x}$$

$$= \frac{2\sec x(\sec x + 1)}{(1+\sec x)\tan x}$$

$$= \frac{2\sec x}{\tan x}$$

$$= \frac{2\left(\frac{1}{\cos x}\right)}{\left(\frac{\sin x}{\cos x}\right)} = \frac{2}{\sin x} = \boxed{2\csc x}$$

4. $\cos^4 x - \sin^4 x = 1 - 2\sin^2 x$

$$= (1-\sin^2 x)^2 - \sin^4 x$$

$$= (1 - 2\sin^2 x + \sin^4 x) - \sin^4 x$$

$$= \boxed{1 - 2\sin^2 x}$$

5. $\sec^2 x + \csc^2 x = \sec^2 x \csc^2 x$

$= (1 + \tan^2 x) + (1 + \cot^2 x)$

$= \left(1 + \frac{\sin^2 x}{\cos^2 x}\right) + \left(1 + \frac{\cos^2 x}{\sin^2 x}\right)$

$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} + \frac{\sin^2 x + \cos^2 x}{\sin^2 x}$

$= \frac{\sin^2 x \cos^2 x + \sin^4 x + \sin^2 x \cos^2 x + \cos^4 x}{\cos^2 x \sin^2 x}$

$= \frac{2\sin^2 x \cos^2 x + \sin^4 x + \cos^4 x}{\cos^2 x \sin^2 x}$

$= \frac{(\sin^2 x + \cos^2 x)^2}{\cos^2 x \sin^2 x}$

$= \frac{(1)^2}{\cos^2 x \sin^2 x} = \frac{1}{\cos^2 x \sin^2 x} = \boxed{\sec^2 x \csc^2 x}$

6. $\frac{1 - \sin^2 x}{1 + \cot^2 x} = \sin^2 x \cos^2 x$

$= \frac{\cos^2 x}{\csc^2 x}$

$= \cos^2 x \cdot \frac{1}{\csc^2 x}$

$= \boxed{\cos^2 x \cdot \sin^2 x}$

7. $\frac{\tan^2 x}{1 + \tan^2 x} = \sin^2 x$

$= \frac{\tan^2 x}{\sec^2 x}$

$= \frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos^2 x}}$

$= \frac{\sin^2 x}{\cos^2 x} \cdot \frac{\cos^2 x}{1}$

$= \boxed{\sin^2 x}$

8. $\frac{\cos x}{1 + \sin x} + \frac{1 + \sin x}{\cos x} = 2\sec x$

$= \frac{\cos^2 x + 1 + 2\sin x + \sin^2 x}{(1 + \sin x)(\cos x)}$

$= \frac{1 + 1 + 2\sin x}{(1 + \sin x)\cos x}$

$= \frac{2 + 2\sin x}{(1 + \sin x)\cos x}$

$= \frac{2(1 + \sin x)}{(1 + \sin x)\cos x}$

$= \frac{2}{\cos x} = \boxed{2\sec x}$