Name: ____________________

14.3 Verify Trigonometric Identities

Trigonometric Identity – A trig equation that is true for all values of the variable θ.

Fundamental Trigonometric Identities

Reciprocal Identities

$\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

Tangent and Cotangent Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

*****Pythagorean Identities*****

$\sin^2\theta + \cos^2\theta = 1$

$1 - \cos^2 x = \sin^2 x$

$-\sin^2 x = -1 + \cos^2 x$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

Cofunction Identities

$\sin(\frac{\pi}{2} - \theta) = \cos\theta$

$\cos(\frac{\pi}{2} - \theta) = \sin\theta$

$\tan(\frac{\pi}{2} - \theta) = \cot\theta$

$\sec(\frac{\pi}{2} - \theta) = \csc\theta$

Negative Angle Identities

$\sin(-\theta) = -\sin\theta$

$\tan(-\theta) = -\tan\theta$

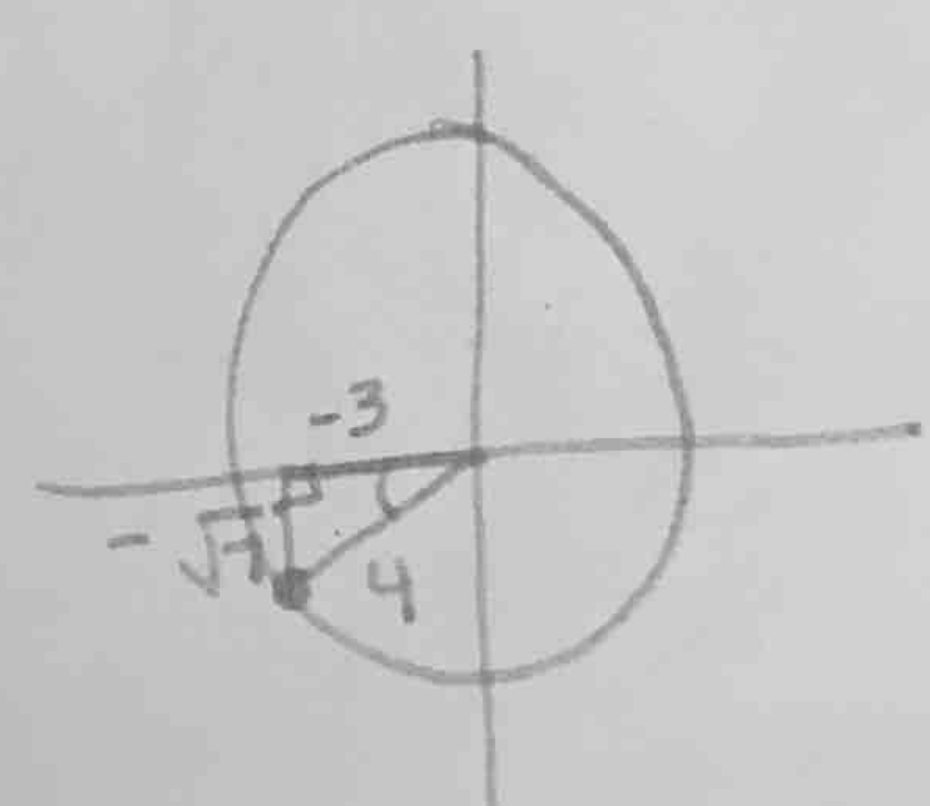
$\cos(-\theta) = \cos\theta$

Ex 1 – Given that $\cos\theta = \frac{-3}{4}$ and $\pi < \theta < \frac{3\pi}{2}$ find the values of the other 5 trig functions of θ.

$\sec\theta = \frac{-4}{3}$

$\sin\theta = \frac{-\sqrt{7}}{4}$ $\csc\theta = \frac{-4\sqrt{7}}{7}$

$\tan\theta = \frac{\sqrt{7}}{3}$ $\cot\theta = \frac{3\sqrt{7}}{7}$



Ex 2 – Simplify the expression: $\dfrac{1}{\sin(\frac{\pi}{2}-\theta)} \cdot \cot\theta$

$= \dfrac{1}{\cos\theta} \cdot \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{1}{\sin\theta}$

$= \csc\theta$

Ex 3 – Simplify the expression: $\dfrac{\tan\theta}{\sec\theta} \cdot \sin\theta + \tan\theta \cdot \csc\theta \cdot \cos^3\theta$

$= \dfrac{\frac{\sin\theta}{\cancel{\cos\theta}}}{\frac{1}{\cancel{\cos\theta}}} \cdot \sin\theta + \dfrac{\cancel{\sin\theta}}{\cos\theta} \cdot \dfrac{1}{\cancel{\sin\theta}} \cdot \cos^3\theta$

$= \sin^2\theta + \cos^2\theta$

$= \boxed{1}$

Ex 4 – Verify the identity.

$\sec\theta \cdot \dfrac{1}{\cos\theta} - \tan\theta \cdot \cot\theta = \dfrac{1-\cos^2\theta}{1-\sin^2\theta}$

$\dfrac{1}{\cos\theta} \cdot \dfrac{1}{\cos\theta} - \tan\theta \cdot \dfrac{1}{\tan\theta}$

$\dfrac{1}{\cos^2\theta} - 1 = \dfrac{1}{\cos^2\theta} - \dfrac{\cos^2\theta}{\cos^2\theta} = \dfrac{1-\cos^2\theta}{\cos^2\theta} = \boxed{\dfrac{1-\cos^2\theta}{1-\sin^2\theta}}$ ✓

HOMEWORK

P 927

14.3 pg. 927 #3-~~33~~ 19 odd, 39 25-35 odd

3.) $\left(\frac{1}{3}\right)^2 + \cos^2\theta = 1$ $\quad \tan\theta = \frac{\frac{1}{3}}{\frac{2\sqrt{2}}{3}} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$

$\cos^2\theta = 1 - \frac{1}{9}$

$\cos^2\theta = \frac{8}{9}$

$\boxed{\cos\theta = \pm\frac{2\sqrt{2}}{3}}$ $\boxed{\tan\theta = \frac{\sqrt{2}}{4}}$ $\boxed{\csc\theta = 3}$ $\boxed{\sec\theta = \frac{3\sqrt{2}}{4}}$ $\boxed{\cot\theta = 2\sqrt{2}}$

5.) $\sin^2\theta + \left(\frac{5}{6}\right)^2 = 1$ $\quad \tan\theta = \frac{\frac{-\sqrt{11}}{6}}{\frac{5}{6}} = \frac{-\sqrt{11}}{5}$

$\sin^2\theta = \frac{11}{36}$

$\boxed{\sin\theta = \frac{-\sqrt{11}}{6}, \tan\theta = -\frac{\sqrt{11}}{5}, \csc\theta = -\frac{6\sqrt{11}}{11}, \sec\theta = \frac{6}{5}, \cot\theta = \frac{-5\sqrt{11}}{11}}$

7.) $1 + \left(-\frac{2}{5}\right)^2 = \csc^2\theta$ $\quad 1 + \left(-\frac{5}{2}\right) = \sec^2\theta$

$\frac{29}{4} = \sec^2\theta$

$\frac{29}{25} = \csc^2\theta$ $\quad -\frac{\sqrt{29}}{2} = \sec\theta$

$\boxed{\frac{\sqrt{29}}{5} = \csc\theta, \sin\theta = \frac{5\sqrt{29}}{29}, \tan\theta = -\frac{5}{2}, \sec\theta = -\frac{\sqrt{29}}{2}, \cos\theta = -\frac{2\sqrt{29}}{29}}$

9.) $1 + \cot^2\theta = \frac{9}{4}$

$\cot^2\theta = \frac{5}{4}$

$\cot\theta = \frac{\sqrt{5}}{2}$

$\tan\theta = \frac{-2\sqrt{5}}{5}$

A

11.) $-\tan\theta$

13.) $\cos\theta \cdot \sec^2\theta =$

$\frac{1}{\sec\theta} \cdot \sec^2\theta = \boxed{\sec\theta}$

17.) $\cos\theta \cdot \frac{1}{\cos\theta} = \boxed{1}$

15.) $\sin x \cdot \sin x = \boxed{\sin^2 x}$

19.) $\frac{\sec x \sin x + \sin x}{1 + \sec x} = \frac{\sin x(\sec x + 1)}{\sec x + 1} = \boxed{\sin x}$

25.) $\sin x \cdot \frac{1}{\sin x} = 1$.

$1 = 1$ ✓

27.) $\frac{\sin\theta + 1}{1 - -\sin\theta} = 1$

$\frac{\sin\theta + 1}{1 + \sin\theta} = 1$

$1 = 1$ ✓

29.) $\frac{1 + \cot^2\theta - \cot^2\theta}{\cos^2\theta} = \sec^2\theta$

$\frac{1}{\cos^2\theta} = \sec^2\theta$

$\sec^2\theta = \sec^2\theta$ ✓

31.) $\sin x + \cos x \cdot \frac{\cos x}{\sin x} = \csc x$

$\sin x + \frac{\cos^2 x}{\sin x} = \csc x$

$\frac{\sin^2 x + \cos^2 x}{\sin x} = \csc x$

$\frac{1}{\sin x} = \csc x$

$\csc x = \csc x$ ✓

33.) $\frac{1 + 2\cos x + \cos^2 x + \sin^2 x}{\sin x\,(1+\cos x)} = 2\csc x$

$\frac{1 + 2\cos x + 1}{\sin x\,(1+\cos x)} = 2\csc x$

$\frac{2\cancel{(1+\cos x)}}{\sin x\cancel{(1+\cos x)}} = 2\csc x$

$\frac{2}{\sin x} = 2\csc x$

$2\csc x = 2\csc x$ ✓

35.) ODD: $\sin x$, $\csc x$, $\tan x$, $\cot x$

EVEN: $\cos x$, $\sec x$

39.) $\sec x \tan x - \sin x = \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} - \sin x = \frac{\sin x}{\cos^2 x} - \frac{\sin x \cos^2 x}{\cos^2 x}$

$= \frac{\sin x(1-\cos^2 x)}{\cos^2 x} = \frac{\sin x \cdot \sin^2 x}{\cos^2 x} = \sin x \cdot \tan^2 x$ ✓